Comment on "Eigenvector Derivatives with Repeated Eigenvalues"

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A recent article by Dailey[1] derives the relations which were previously published in Ref. 2. Both works correct Ojalvo's[3] repeated root extensions to Nelson's eigenvector sensitivity method.[4] However, Dailey reproduces the same error that appeared in Ref. 3 regarding the selection of submatrices. This error was corrected in Ref. 2.

A critical step in Dailey's method may fail under certain circumstances. The step which may fail is the decomposition of a partition of the singular system

$$K - \lambda M \tag{1}$$

This is step 6 of Dailey's method. The ability to decompose a partition of these matrices is dependent upon the appropriate selection of submatrices (Dailey's step 5, which was originally suggested by Ojalvo.[3])

A small example serves to illustrate the problem. The stiffness and mass matrices and their sensitivities are

$$K = \begin{bmatrix} 3 & -2 & -2 \\ -2 & 2 & 0 \\ -2 & 0 & 8 \end{bmatrix}; \quad M = \begin{bmatrix} 2 & -1 & -2 \\ -1 & 1 & 0 \\ -2 & 0 & 8 \end{bmatrix} \tag{2}$$

$$K' = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 16 \end{bmatrix}; \quad M' = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 4 \end{bmatrix} \tag{3}$$

The eigensolution is

$$X = \begin{bmatrix} 1.0 & 1.0 & 0.0 \\ 1.0 & 1.0 & 1.0 \\ 0.0 & 0.5 & 0.0 \end{bmatrix}; \quad \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{4}$$

The eigenvalue sensitivities are

$$\Lambda' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{5}$$

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For the first two (repeated) roots

$$G \equiv K - \lambda M = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

Following the directions in step 5, rows and columns 1 and 2 are set to zero, and their diagonal elements are set to one.

$$\bar{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$\bar{G}$ is clearly singular, and cannot be used to solve the equations in step 6.

Reference 2 had previously provided a more rigorous basis for modifying the G matrix and contained the development of the subject matter presented by Dailey. The selection of rows and columns of G for modification is based on avoiding singularities in the corresponding rows of the repeated-root eigenvectors. Using this criterion, two different sets of rows and columns are guaranteed to generate a nonsingular $\bar{G}$ matrix. These two sets are rows 1 and 3 and rows 2 and 3. For either set

$$\bar{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$\bar{G}$ is clearly nonsingular.

For a small problem, as the one shown here, failure to generate a nonsingular $\bar{G}$ is not serious—a failure can be followed by a reselection of rows and columns until a nonsingular $\bar{G}$ is found. However, for large systems the singularity is detected only after $\bar{G}$ has been decomposed, which is the most expensive step in the solution. Thus, it is important to use the rigorous method for the calculation of $\bar{G}$.

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References

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[3]Ojalvo, I. U., "Gradients for Large Structural Models with Repeated Frequencies," SAE Paper 86-1789, Oct. 1986.

[4]Nelson, R. B., "Simplified Calculation of Eigenvector Derivatives," *AIAA Journal,* Vol. 14, Sept. 1976, pp. 1201–1205.